Bayesian Statistics

By: Drake Scott

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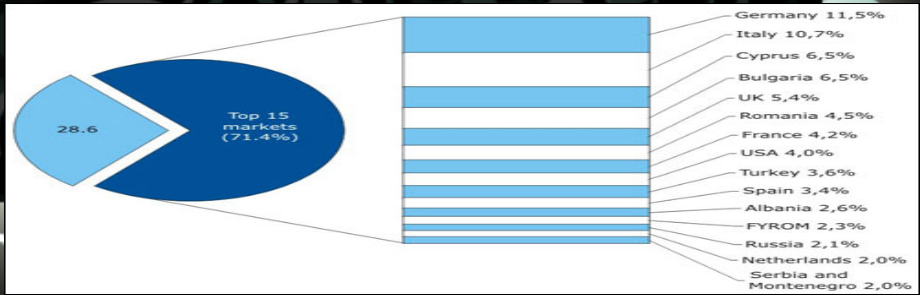

The Beginning: Reverend Thomas Bayes (1702 - 1761)

- The Reverend devised Bayes' Theorem in his published work *An Essay towards solving a Problem in the Doctrine of Chances.*

Reverend Thomas Bayes

- Bayes' Theorem studies the correlation of events rather than their order.
- The Reverend focused much time on computing the distribution for a parameter of a binomial distribution.

Pierre-Simon Laplace (1749-1827)

- One of the founding fathers of Bayesian statistical analysis.
- Introduced a general version of Bayes' Theorem.
- He the applied his methods to problems in celestial mechanics, medical statistics, and reliability.

Beyond the Mystery: Bayes' Theorem

- Bayes' Theorem is a method of statistical inference, it is used to infer the probability of a hypothesis based on the events and observations.
- The model is considered to be a hidden variable with a prior distribution.
- The theorem relates conditional probabilities.
- If two events are denoted by A and B, then the conditional probability of A given that B occurs is equal to P(A | B)
- The two conditional probabilities are related through Bayes' Theorem, P(A | B)*P(B)=P(B | A)*P(A)

Derivation of Theorem from Conditional Probabilities

- P(A|B) = P(AПB) / P(B) and
  P(B|A) = P(AПB) / P(A)
- P(A|B) * P(B) = P(B|A) * P(A) = P(AПB)
- Just set the equation up and divide by P(B)
- P(A|B) = P(B|A)*P(A) / P(B) <-Bayes' Theorem

Some Examples of its use:

- Conditional probabilities

There are two giant top hats. Hat #1 contains 20 plastic pandas and 30 model anteaters, #2 contains 40 plastic pandas and 10 model anteaters.

- A hat and item are chosen at random, so happens a plastic panda is snagged out of one of the giant hats.
- Find the probability that it was snagged from hat #1, given a plastic panda is chosen.

A = hat #1 P(A) = 0.5

B = p. panda P(B) = 0.6

P(B|A) = 0.4

- P(A|B) = (P(B|A)*P(A)) / (P(B)) = (0.4*0.5) / (0.6) = 0.333

Another example:

- False positive in a medical test.
- A = having the disease   P(A) = 0.05
- B = positive test   Ac = not having the disease
- If one has the disease, they test positive with a P(B|A) = 0.95
- If one does not have the disease, they test positive with a P(B|Ac) = 0.01
- P(A|B) = (P(B|A)*P(A)) / (P(B|A)*P(A) + P(B|Ac)*P(Ac))

= (0.95*0.05) / (0.95*0.05 + 0.01*0.99)

- = 0.8275 = P(A|B) = probability that a positive result will not be a false positive.

The End